2020-2021

KARNATAKA LAW SOCIETY’S

GOGTE INSTITUTION OF TECHNOLOGY, BELAGAVI

DEPARTMENT OF MATHS



A PROJECT ON

**“NUMERICAL ANALYSIS”**

SUBMITTED BY

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INTRODUCTION

*The great advantage of using numerical analysis is that it investigates and provides accurate solutions to real-life problems from the field of science, engineering, biology, astrophysics and finance.*

The word ‘analysis’ generally means to solve a problem through a set of equations and further reduce these equations using the methodologies of algebra, partial differential equations, calculus and other related fields of mathematics.

On similar grounds, numerical analysis implements arithmetic algorithms: addition, subtraction, multiplication, division and comparison to obtain numerical solutions.

A computer precisely performs these operations, meaning that numerical analysis and computers are intimately related.

The problem of continuous mathematics generally arises throughout the natural sciences, business management, engineering, astronomy and medicine. It is always more practical to carry out the tedious arithmetic operations using a computer.

Before the mid-20th century, until the advent of modern computers, all these repetitive operations had to be performed by manual interpolation. The overall agenda of numerical analysis is to give an *approximate*, but *accurate* solution to the advanced problem. The necessity for accuracy, is, of course, determined by the variety of applications.

## *Historical background*

Numerical algorithms predate the invention of modern and digital computers and are almost as sold as human civilization itself. Many renowned mathematicians of the past were engrossed in numerical analysis. Archimedes of Syracuse (287-211/212 BC), the most famous Greek mathematician, physicist, engineer and inventor, perfected new standards of contemporary geometry, including the ‘method of exhaustion‘ to compute the area and volume of various geometric figures.

A famous document, The Rhind Papyrus (1650 BC) from the ancient Egyptian Middle Kingdom, bears a root-finding method to solve simple equations.

To facilitate computational solutions, Newton and Leibnitz invented integral and differential calculus. Various accurate mathematical models were developed, but they couldn’t be solved explicitly

To simplify this, several numerical methods were created by Isaac Newton. Following his work on root finding and interpolations, many other legendary mathematicians like Euler, Lagrange, and Gauss contributed to the field of numerical analysis. Another important aspect was the invention of logarithms by John Naiper (1614), replacing the tedious calculations of multiplication, division and exponentiation.

## Direct and iterative methods

Direct methods compute the solution to a problem in a finite number of steps. These methods would give the precise answer if they were performed in infinite precision arithmetic. Examples include Gaussian elimination, the QR factorization method for solving systems of linear equations, and the simplex method of linear programming. In practice, finite precision is used and the result is an approximation of the true solution (assuming stability).

In contrast to direct methods, iterative methods are not expected to terminate in a number of steps. Starting from an initial guess, iterative methods form successive approximations that converge to the exact solution only in the limit. A convergence test is specified in order to decide when a sufficiently accurate solution has (hopefully) been found. Even using infinite precision arithmetic these methods would not reach the solution within a finite number of steps (in general). Examples include Newton’s method, the bisection method, and Jacobi iteration. In computational matrix algebra, iterative methods are generally needed for large problems.

Iterative methods are more common than direct methods in numerical analysis. Some methods are direct in principle but are usually used as though they were not, e.g. GMRES and the conjugate gradient method. For these methods the number of steps needed to obtain the exact solution is so large that an approximation is accepted in the same manner as for an iterative method.

## Discretization

Furthermore, continuous problems must sometimes be replaced by a discrete problem whose solution is known to approximate that of the continuous problem; this process is called discretization. For example, the solution of a differential equation is a function. This function must be represented by a finite amount of data, for instance by its value at a finite number of points at its domain, even though this domain is a continuum.

Different methods and areas under Numerical Analysis**:** Direct methods lead us to the exact solution in a finite number of steps. For example, Gauss elimination is used to find the roots of the linear simultaneous equations immediately. On the contrary, iterative methods are most commonly used and are not expected to finish off in a certain number of steps. These are approximate methods starting from an initial guess and converging to an exact solution. Gauss- Jacobi and Gauss- Seidel are iterative methods used to solve a system of equations with a larger number of unknowns.

According to the problem to be solved, the field of numerical analysis is divided into various disciplines.

* Interpolation constructs a new set of data points within the range of the given function (or problem). This can be obtained through curve-fitting, and some examples include the Gaussian process, linear interpolation and polynomial interpolation.
* Extrapolation is identical to interpolation, but it has a higher risk of producing meaningless results. It is the process of roughly evaluating the value of the problem outside its range.
* Regression is a statistical process that helps in understanding the relationship between dependent and independent variables. It is widely used for forecasting and predicting in the field of machine learning.
* Solving differential and integral equations:  Most mathematical models (particularly in engineering) are based on the solutions obtained by partial differential equations, ordinary differential equations and integral equations. Some popular techniques are Monte Carlo integration and the Newton-Cotes formula.

## Modern applications of Numerical Analysis

Sophisticated numerical analysis software has become indispensable in modern life. People are able to perform mathematical modeling even if they are unaware of the simulations involved. This can only be achieved through reliable, high-end and efficient software. Some of the major applications of numerical analysis are intriguing, yet easy to understand.

* Car safety enhancements: Car makers around the globe use numerical simulations to evaluate and enhance car safety. Pedestrian protection is also kept in mind while investigating car crash tests. The algorithms involved are partial differential equations and fed to the advanced computers to unravel optimal results.
* *Airflow patterns in the Respiratory Tract:*It is quite common for patients in ICUs to undergo respiratory failure. Mechanical [ventilation](https://www.semanticscholar.org/paper/Optimal-Determination-of-Respiratory-Airflow-for-a-Hou-Meskin/07c3ee2d57474f15f511945eeb666b794f7d7899?p2df) is a treatment that helps in the sufficient exchange of oxygen and carbon dioxide for the normal functioning of the lungs. Various mathematical models use differential equations and computational algorithms to develop laminar airflow in the lungs using ventilators.
* *Onset and Progression of tumor cells:*Cancer is characterized by the accumulation of pre-malignant cells and tumor growth. In recent years, cost-effective statistical and probability models have been developed to detect cancer in the body. This allows for the computing of several parameters, such as population size, lifetime and mutations of the cancer cells.
* *Financial industry*: Modern businessmen make use of numerical techniques to allocate their resources efficiently. Some of the problems addressed by such applications are manufacturing, storage, scheduling, investment and others. Quantitative analysts have expertise in this area, and use the algorithms in risk management and interest calculation.
* *Transportation of chemicals in the body:* Our body is constantly being exposed to various chemicals (or drugs). Not all are beneficial and the body must excrete them out into the environment. The [diffusion](https://www.researchgate.net/publication/226787826_Modeling_Transport_Processes_and_Their_Implications_for_Chemical_Disposition_and_Action) and transport of such chemicals is studied with the help of ordinary and partial differential equations.
* *High hydrostatic pressure (HHP) processing:* This is a non-thermal process in which food and biotechnological substances are compressed under very high pressure of up to 1000 MPa to inactivate certain enzymes and micro-organisms. The treatment of fluid food is analyzed by means of numerical simulations. The enzyme is inactivated with the help of numerical equations.
* *Weather predictions:*Numerical weather predictions (NWP) are based on a set of differential equations known as hydro-thermodynamic equations. Very powerful and energy-efficient computers are used to process the bulk data and the information is extracted in the form of topographical charts.
* *Spacecraft Dynamics*: Increases in the size and complexity of spacecraft have demanded a complex mathematical model of its dynamics. To reduce the inconvenience and plan a smooth trajectory for the spacecraft, various open loop models are created because the dynamics in space behave very differently than they do on Earth.
* Price estimation by airlines: Nowadays, airline ticket prices vary significantly, even for nearby seats within the same cabin. Airlines use computational techniques to increase their revenue, keeping a check on fuel, payroll, crew assignments and many other activities.

Machine learning: The numerical algorithms of Newton’s method and the Nestorov method are used in machine learning optimization.

## *Optimization*

Optimization problems ask for the point at which a given function is maximized (or minimized). Often, the point also has to satisfy some constraints.

The field of optimization is further split in several subfields, depending on the form of the objective function and the constraint. For instance, linear programming deals with the case that both the objective function and the constraints are linear. A famous method in linear programming is the simplex method.

The method of Lagrange multipliers can be used to reduce optimization problems with constraints to unconstrained optimization problems.

## Evaluating integrals

Numerical integration, in some instances also known as numerical quadrature, asks for the value of a definite integral. Popular methods use one of the Newton-Cotes formulas (like the midpoint rule or Simpson’s rule) or Gaussian quadrature. These methods rely on a “divide and conquer” strategy, whereby an integral on a relatively large set is broken down into integrals on smaller sets. In higher dimensions, where these methods become prohibitively expensive in terms of computational effort, one may use Monte Carlo or quasi-Monte Carlo methods (see Monte Carlo integration), or, in modestly large dimensions, the method of sparse grids.

## Differential equations

Numerical analysis is also concerned with computing (in an approximate way) the solution of differential equations, both ordinary differential equations and partial differential equations.

Partial differential equations are solved by first discretizing the equation, bringing it into a finite-dimensional subspace. This can be done by a finite element method, a finite difference method, or (particularly in engineering) a finite volume method. The theoretical justification of these methods often involves theorems from functional analysis. This reduces the problem to the solution of an algebraic equation.

## Applications Of Numerical Analysis Methods and Its Real Life Implementations, Advantages

### *****NEWTON RAPHSON METHOD:*****

ORDER OF CONVERGENCE: 2 ADVANTAGES: 1. The advantage of the method is its order of convergence is quadratic. 2. Convergence rate is one of the fastest when it does converges 3. Linear convergence near multiple roots.

**REGULA FALSI METHOD: ORDER OF CONVERGENCE:**

**ADVANTAGES:**

1. Better-than-linear convergence near simple root

2. Linear convergence near multiple root

3. No derivative needed DISADVANTAGES

1. Iterates may diverge 2. No practical & rigorous error bound

## GAUSS ELIMINATION METHOD:

## ADVANTAGES:

It is the direct method of solving linear simultaneous equations. 2. It uses back substitution. 3. It is reduced to equivalent upper triangular matrix.: 1. It requires right vectors to be known.

**GAUSS JORDAN: ADVANTAGES:**

1. It is direct method.

2. The roots of the equation are found immediately without using back substitution.

3 It is reduced to equivalent identity matrix.

4. The additional steps increase round off errors.

5. It requires right vectors to be known.

## GAUSS JACOBI METHOD:

1. It is iterative method. 2. The system of equations must be diagonally dominant. 3. It suits better for large numbers of unknowns 4. It is self correcting method.

## GAUSS SEIDEL METHOD:

1. It is iterative method. 2. The system of equations must be diagonally dominant. 3. It suits better for large numbers of unknowns 4. It is self correcting method. 5. The number of iterations is less than Jacobi method.

*REAL LIFE USES OF ANALYSIS METHOD*

## Most powerful tools of numerical analysis

1.Computer graphics

2.Symbolic mathematical computations

3.Graphical user interfaces

Numerical analysis is needed to solve engineering problems that lead to equations that cannot be solved analytically with simple formulas.

Examples are solutions of large systems of algebraic equations, evaluation of integrals, and solution of differential equations. The finite element method is a numerical method that is in widespread use to solve partial differential equations in a variety of engineering fields including stress analysis, fluid dynamics, heat transfer, and electro-magnetic fields. In Hydro Static Pressure Processing

In high hydrostatic pressure (HHP) processing, food and biotechnological substances are compressed up to 1000 M Pa to achieve various pressure-induced conversions such as microbial and enzyme inactivation’s, phase transitions of proteins, and solid-liquid state transitions.

From the point of view of thermodynamics, Heat transfer leads to space-time-dependent temperature fields that affect many pressure-induced conversions and produce undesired process non uniformities

Effects related to HHP processing can be studied appropriately by use of numerical analysis because in situ measurement techniques are barely available, optical accessibility is hardly possible, and technical equipment is expensive.

This reports on two examples, where numerical analysis is applied successfully and delivers substantial insights into the phenomenon of high-pressure processing.

## Calculation

E.g TSP problem (traveling salesman problem)

to travel no. of cities in such a way that the expenses on traveling are minimized.

à NP-complete problem.

à optimal solution we have to go through all possible routes

à numbers of routes increases exponential with the numbers of cities.

## Modern Applications and Computer Software

Sophisticated numerical analysis software is being embedded in popular software packages **e.g.** spreadsheet programs.

## Buisness Applications:-

Modern business makes much use of optimization methods in deciding how to allocate resources most efficiently. These include problems such as inventory control,scheduling, how best to locate manufacturing storage facilities, investment strategies,and others.

## In Financial Industry

Quantitative analysts developing financial applications have specialized expertise in their area of analysis.

Algorithms used for numerical analysis range from basic numerical functions to calculate interest income to advanced functions that offer specialized optimization and forecasting techniques.

## Sample Finance Applications

Three common examples from the financial services industry that require numerical algorithms are:

\*Portfolio selection

\* Option pricing

\* Risk management

## In market

Given the broad range of numerical tools available a financial services provider can develop targeted applications that address specific market needs. For example, quantitative analysts developing financial applications have specialized expertise in their area of analysis

## Conclusion

Numerical analysis is the branch of modern computation that finds applications in the field of engineering, life sciences and even arts. It has a remarkable ability to predict the world around us. The calculations are mostly made by the computers using MATLAB, FORTRAN 77 and other software programs to minimize errors. Clearly, numerical analysis has proved itself as a boon to humankind, from ancient times all the way to today, and they will surely help us move forward into the future!

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THANK YOU